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# QCD susceptibilities and nuclear-matter saturation in a relativistic chiral theory

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**Abstract.** We investigate the evolutions with density of the QCD scalar susceptibility and of the sigma mass in a chiral relativistic theory of nuclear matter, in the mean-field approximation. In order to reach saturation we need to introduce the scalar response of the nucleons. The consequences are a quite mild density dependence of the sigma mass and the progressive decoupling of the quark density fluctuations from the nucleonic ones at large densities.

**PACS.** 24.85.+p Quarks, gluons, and QCD in nuclei and nuclear processes – 11.30.Rd Chiral symmetries – 13.75.Cs Nucleon-nucleon interactions (including antinucleons, deuterons, etc.) – 21.30.-x Nuclear forces

## 1 Introduction

The order parameter associated with spontaneous breaking of chiral symmetry, namely the chiral quark condensate, is influenced in the nuclear medium by the mean scalar field which possesses the same quantum numbers [1]. Its fluctuations in the medium are intimately related as well to the in-medium propagation of the scalar field [2]. On the other hand, the properties of the scalar-field influence in a crucial way the question of the nuclear binding. It is therefore important for these problems to describe the nuclear dynamics in a way which satisfies the chiral constraints and is able to correctly reproduce the binding and saturation properties. The key ingredients can be found in the theory of quantum hadrodynamics (QHD) [3,4], in the chiral version of ref. [1]. However, there is a well-identified problem concerning the nuclear saturation with usual chiral effective theories [5–7]. Independently of the particular chiral model, in the nuclear medium one moves away from the minimum of the vacuum effective potential (Mexican-hat potential), *i.e.*, into a region of smaller curvature. This single effect, equivalent to the lowering of the sigma mass, destroys the stability, creating problems for the applicability of such effective theories in the nuclear context. One possible way to cure this problem introduces the nucleonic response to the scalar field, $\kappa_{NS}$, which is the central ingredient of the quark-meson coupling model, QMC [8,9], which is quite successful in the phenomenology of nuclear matter or finite nuclei. Indeed, the introduction of $\kappa_{NS}$ which has a positive sign implies a density dependence of the scalar nucleon coupling constant, which actually decreases with increasing density. This effect can counterbalance the decrease of the sigma mass and restore saturation. It is the aim of this work to explore the density evolution of the scalar meson mass and of the QCD scalar susceptibility in a realistic chiral effective theory which incorporates the concept of a nucleonic scalar response to a scalar field. Inside our framework, *i.e.*, the sigma model formulated in a non-linear version but with the presence of a chiral singlet scalar field, we will choose the parameters so as to be compatible with the known saturation properties of nuclear matter: saturation density, binding energy and compression modulus. In our fit of the nuclear-matter data $\kappa_{NS}$ is a parameter that we can adjust. But the sign found in QMC is crucial. It originates from relativistic effects of quarks confined in a bag ($Z$-graphs) and it is positive. It screens the scalar field (diamagnetic effect). The fact that the nucleonic scalar response manifests itself at the nuclear level is plausible and familiar in other situations. For instance, under the influence of an isovector axial field, the nucleon converts into a $\Delta$. This axial polarizability of the nucleon has several physical implications (such as the in-medium renormalization of the axial coupling constant).

The problem of the in-medium sigma mass is interesting in connection with two-pion production experiments. A lowering of this mass due to the partial restoration of chiral symmetry, as we have previously discussed, has been proposed in [10] as the origin of the accumulation of

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strength near the two-pion threshold in these experiments on nuclei [11–13]. Since the pure chiral effective theories with the full dropping of the sigma mass are incompatible with saturation, it is interesting to explore what modification is allowed in order to be compatible with the nuclear-matter data. The result of our investigation is that the density dependence of the sigma mass has to be quite mild.

Section 2 is devoted to the formal derivation from the equation of state of various quantities: sigma mass, QCD susceptibility, nuclear response. In sect. 3 we perform the numerical evaluations of these quantities, ensuring the compatibility with the nuclear phenomenology and we discuss the consequences.

## 2 Equation of state and scalar fluctuations

We start with the Lagrangian introduced in our previous work [1], dropping its pionic part which does not contribute at the mean-field level:

$$\mathcal{L} = \frac{1}{2}\partial^\mu s \partial_\mu s - V(s) + i\bar{N}\gamma^\mu\partial_\mu N - M_N\left(1+\frac{s}{f_\pi}\right)\bar{N}N\,, \tag{1}$$

where $s$ is the chiral invariant field associated with the radius $S = s + f_\pi$ of the chiral circle. $V(s) = V_0(s) - cS$ is the vacuum potential which can be split into $V_0(s) = (\lambda/4)\big((f_\pi+s)^2 - v^2\big)^2$ responsible for spontaneous chiral-symmetry breaking and the explicit symmetry breaking piece, $-cS$, where $c = f_\pi m_\pi^2$. As usual, in QHD we add a coupling to an omega field:

$$\mathcal{L}_V = \frac{1}{2}\partial^\mu\omega\partial_\mu\omega + \frac{1}{2}m_\omega^2\,\omega^2 - g_\omega\,\omega\,N^\dagger N. \tag{2}$$

As for the nucleonic response, $\kappa_{NS}$, to the scalar field $s$, we incorporate it in the following extra term in the Lagrangian:

$$\mathcal{L}_\chi = -\frac{1}{2}\,\kappa_{NS}\,s^2\,\bar{N}N. \tag{3}$$

The susceptibility $\kappa_{NS}$ embeds the influence of the internal nucleon structure. We will discuss later its possible $s$-dependence. At the mean-field level the energy density is given by

$$\varepsilon = \int\frac{4\,\mathrm{d}^3p}{(2\pi)^3}\,\Theta(p_F-p)\,E_p^*(\bar{s}) + V(\bar{s}) + \frac{g_\omega^2}{2\,m_\omega^2}\,\rho^2\,, \tag{4}$$

where the baryonic density is related to the Fermi momentum through

$$\rho = \int\frac{4\,\mathrm{d}^3p}{(2\pi)^3}\,\Theta(p_F-p) \tag{5}$$

and $E_p^*(\bar{s}) = \sqrt{p^2 + M_N^{*2}(\bar{s})}$ is the energy of an effective nucleon with the effective mass

$$M_N^* = M_N\left(1+\frac{\bar{s}}{f_\pi}\right) + \frac{1}{2}\,\kappa_{NS}\,\bar{s}^2. \tag{6}$$

The expectation value, $\bar{S} = f_\pi + \bar{s}$, of the $S$-field, plays the role of the chiral order parameter. It is obtained by minimizing the energy density,

$$\frac{\partial\varepsilon}{\partial\bar{s}} = g_S^*\,\rho_S + V'(\bar{s}) = 0\,, \tag{7}$$

with the following expressions for the scalar density, $\rho_S$, and the scalar coupling constant $g_S^*$:

$$\rho_S = \int\frac{4\,\mathrm{d}^3p}{(2\pi)^3}\,\Theta(p_F-p)\,\frac{M_N^*}{E_p^*}$$

and

$$g_S^*(\bar{s}) = \frac{\partial M_N^*}{\partial\bar{s}} = \frac{M_N}{f_\pi} + \kappa_{NS}\,\bar{s}.$$

Notice that the density dependence of $g_S^*$ entirely arises from the susceptibility term. Since the mean scalar field is negative and the sign of $\kappa_{NS}$ positive, $g_S^*$ is a decreasing function of the density. In the vacuum, the scalar coupling constant of the model is $g_S = M_N/f_\pi$. The in-medium sigma mass is obtained as the second derivative of the energy density with respect to the order parameter:

$$\begin{aligned} m_\sigma^{*2} &= \frac{\partial^2\varepsilon}{\partial\bar{s}^2} = V''(\bar{s}) + \frac{\partial\,(g_S^*\,\rho_S)}{\partial\bar{s}} = \\ &m_\sigma^2\left(1+\frac{3\bar{s}}{f_\pi}+\frac{3}{2}\left(\frac{\bar{s}}{f_\pi}\right)^2\right) + \kappa_{NS}\,\rho_S + g_S^*\,\frac{\partial\rho_S^*}{\partial\bar{s}}\,, \end{aligned} \tag{8}$$

where in the second line we have taken $V''(\bar{s})$ in the chiral limit. The mean scalar field $\bar{s}$ being negative, the term linear in $\bar{s}$ (which appears from the curvature of the effective potential) in itself lowers the sigma mass by an appreciable amount ($\simeq 30\%$ at $\rho_0$). This is the chiral dropping associated with chiral restoration emphasized by Hatsuda *et al.* [10], which they suggested to be the origin of the strong medium effects found in $2\pi$ production experiments [11–13]. However, the scalar-susceptibility term (second term in $\kappa_{NS}$ on the r.h.s of eq. (8)) counterbalances this in-medium mass dropping. Numerically, in order to reach saturation, its effect has to be important and the cancellation has to be nearly complete, as we will show later. Hence, the sigma mass evolution has to be discussed in connection with saturation properties. In pure quantum hadrodynamics instead, the chiral softening of the sigma mass is ignored and saturation is obtained entirely through the density evolution of the scalar density of nucleons. As for the last term ($g_S^*\,\partial\rho_S^*/\partial\bar{s}$) of eq. (8), it also writes

$$g_S^*\,\frac{\partial\rho_S^*}{\partial\bar{s}} = g_S^{*2}\int\frac{4\,\mathrm{d}^3p}{(2\pi)^3}\,\Theta(p_F-p)\,\frac{p^2}{E_p^{*3}}. \tag{9}$$

We have shown in a previous work [14] that it actually corresponds to the nuclear response associated with $N\bar{N}$ excitation. In practice it is small and it can be omitted.

We will now derive the in-medium chiral condensate and the QCD scalar susceptibility. They are related to the first and second derivatives of the grand potential with respect to the quark mass $m$ at constant chemical potential $\mu$. The baryonic chemical potential is obtained as

$$\mu = \frac{\partial \varepsilon}{\partial \rho} = E_F^* + \frac{g_\omega^2}{m_\omega^2}\rho \qquad \text{with} \qquad E_F^* = \sqrt{p_F^2 + M_N^{*2}(\bar{s})}\,, \tag{10}$$

from which one deduces that the baryonic density is controled by the chemical potential according to

$$\rho = \int \frac{4\,\mathrm{d}^3p}{(2\pi)^3}\,\Theta\left(\mu - E_p^* - \frac{g_\omega^2}{m_\omega^2}\,\rho\right), \tag{11}$$

while the scalar density writes

$$\rho_S = \int \frac{4\,\mathrm{d}^3p}{(2\pi)^3}\,\frac{M_N^*}{E_p^*}\,\Theta\left(\mu - E_p^* - \frac{g_\omega^2}{m_\omega^2}\,\rho\right). \tag{12}$$

The grand potential, which is obtained through a Legendre transform, can be written in the following form:

$$\begin{aligned}\omega(\mu) = \varepsilon - \mu\,\rho = &\\ \int \frac{4\,\mathrm{d}^3p}{(2\pi)^3}&\left(E_p^* + \frac{g_\omega^2}{m_\omega^2}\,\rho - \mu\right)\\ &\times\Theta\left(\mu - E_p^* - \frac{g_\omega^2}{m_\omega^2}\,\rho\right) + V(s) - \frac{g_\omega^2}{2\,m_\omega^2}\,\rho^2.\end{aligned} \tag{13}$$

Notice that the minimization (eq. (7)) can be equivalently obtained from the condition $(\partial\omega/\partial\bar{s})_\mu = 0$. In order to derive the condensate and susceptibility we point out that, in the context of this model, what plays the role of the chiral-symmetry breaking parameter is the quantity $c = f_\pi m_\pi^2$ which enters the symmetry breaking piece of the potential. Hence,

$$\begin{aligned}\langle\bar{q}q\rangle = \frac{1}{2}\left(\frac{\partial\omega}{\partial m}\right)_\mu = \frac{1}{2}\frac{\partial c}{\partial m}\left(\frac{\partial\omega}{\partial c}\right)_\mu = &\\ -\frac{1}{2}\frac{\partial c}{\partial m}\,\bar{S} \simeq \frac{\langle\bar{q}q\rangle_{vac}}{f_\pi}\,\bar{S}\,,&\end{aligned} \tag{14}$$

where we have used the Feynman-Hellman theorem and the explicit expression of $\partial c/\partial m$ given by the model to leading order in the quark mass $m$. Accordingly, the in-medium scalar susceptibility is given by

$$\begin{aligned}\chi_S = \left(\frac{\partial\langle\bar{q}q\rangle}{\partial m}\right)_\mu = -\frac{1}{2}\left(\frac{\partial c}{\partial m}\right)^2\left(\frac{\partial\bar{S}}{\partial c}\right)_\mu \simeq &\\ -2\,\frac{\langle\bar{q}q\rangle_{vac}^2}{f_\pi^2}\left(\frac{\partial\bar{S}}{\partial c}\right)_\mu.&\end{aligned} \tag{15}$$

The derivative $(\partial\bar{S}/\partial c)_\mu$ is obtained by taking the derivative of the minimization equation (7) with respect to the parameter $c$. This gives

$$\begin{aligned}m_\sigma^{*2}\left(\frac{\partial\bar{S}}{\partial c}\right)_\mu &= 1 - g_S^*\,\Pi_0(0)\\ &\times\left[g_S^*\,\frac{M_N^*}{E_F^*}\left(\frac{\partial\bar{S}}{\partial c}\right)_\mu + \frac{g_\omega^2}{m_\omega^2}\left(\frac{\partial\rho}{\partial c}\right)_\mu\right],\end{aligned} \tag{16}$$

with $\Pi_0(0) = -2M_N^*\,p_F/\pi^2$. Notice that $\Pi_0(0)$ is nothing but the non-relativistic free Fermi-gas particle-hole polarization propagator in the Hartree scheme, at zero energy in the limit of the vanishing momentum. The derivative of the baryonic density is obtained by taking the derivative with respect to $c$ of eq. (11), with the result

$$\left(\frac{\partial\rho}{\partial c}\right)_\mu = g_S^*\left(\frac{\partial\bar{S}}{\partial c}\right)_\mu \Pi_0(0)\left(1 - \frac{g_\omega^2}{m_\omega^2}\,\frac{E_F^*}{M_N^*}\,\Pi_0(0)\right)^{-1}. \tag{17}$$

It follows that

$$\left(\frac{\partial\bar{S}}{\partial c}\right)_\mu = \frac{1}{m_\sigma^{*2}} - \frac{1}{m_\sigma^{*2}}\,\Pi_{SS}(0)\,\frac{1}{m_\sigma^{*2}}\,, \tag{18}$$

where $\Pi_{SS}(0)$ is the full scalar polarization propagator (in which we include the coupling constant):

$$\begin{aligned}\Pi_{SS}(0) = g_S^{*2}\,\frac{M_N^*}{E_F^*}\,\Pi_0(0)&\\ \times\left[1 - \left(\frac{g_\omega^2}{m_\omega^2}\,\frac{E_F^*}{M_N^*} - \frac{g_S^{*2}}{m_\sigma^{*2}}\,\frac{M_N^*}{E_F^*}\right)\Pi_0(0)\right]^{-1}.&\end{aligned} \tag{19}$$

We now comment these results. Firstly, notice that the quantity $(\partial\bar{S}/\partial c)_\mu$, as written in eq. (18), is the in-medium scalar meson propagator (up to a minus sign) dressed by $NN^{-1}$ excitations. Moreover, it is satisfactory to realize that the expression of $\Pi_{SS}$ (eq. (19)) coincides with the one derived from the RPA equations in the ring approximation. In RPA the residual interaction can be due either to the scalar meson exchange or to the vector one [15]. In the latter case the mixed propagator $\Pi_{SV}$ enters. The RPA equations read

$$\begin{aligned}\Pi_{SS} &= \Pi_{SS}^0 + \Pi_{SS}^0\,D_S^0\,\Pi_{SS} - \Pi_{SV}^0\,D_V^0\,\Pi_{VS}\,,\\ \Pi_{VS} &= \Pi_{VS}^0 + \Pi_{VS}^0\,D_S^0\,\Pi_{SS} - \Pi_{VV}^0\,D_V^0\,\Pi_{VS}\end{aligned} \tag{20}$$

with $D_S^0 = -1/m_\sigma^{*2}$ and $D_V^0 = -1/m_\omega^2$. The solution for $\Pi_{SS}$ is

$$\Pi_{SS} = \frac{\Pi_{SS}^0 - D_V^0\left(\Pi_{SV}^0\Pi_{VS}^0 - \Pi_{SS}^0\Pi_{VV}^0\right)}{1 - \Pi_{SS}^0 D_S^0 + \Pi_{VV}^0 D_V^0 + D_S^0 D_V^0\left(\Pi_{SV}^0\Pi_{VS}^0 - \Pi_{SS}^0\Pi_{VV}^0\right)}. \tag{21}$$

One recovers our result of eq. (19) since, as shown in the appendix,

$$\begin{aligned}&\Pi_{SS}^0(0) = g_S^{*2}\,\frac{M_N^*}{E_F^*}\,\Pi_0(0), \quad \Pi_{VV}^0(0) = g_\omega^2\,\frac{E_F^*}{M_N^*}\,\Pi_0(0),\\ &\Pi_{SV}^0(0) = \Pi_{VS}^0(0) = g_S^*\,g_\omega\,\Pi_0(0).\end{aligned} \tag{22}$$

In our approach the Landau-Migdal parameter $F_0$ enters the RPA denominator of eq. (19) which, at $\rho_0$, writes $1 + F_0$. This equation also shows that our residual particle-hole interaction is density dependent, in particular through the density dependence of $g_S^*$ and $m_\sigma^*$. Our approach provides a consistent relativistic frame in a chiral theory to derive this density dependence. A similar expression has been given in ref. [6].

As for the full vector polarization propagator $\Pi_{VV}$, solution of the RPA equations, it is given by

$$\Pi_{VV}(0) = g_\omega^2 \frac{E_F^*}{M_N^*} \Pi_0(0) \times \left[1 - \left(\frac{g_\omega^2}{m_\omega^2}\frac{E_F^*}{M_N^*} - \frac{g_S^{*2}}{m_\sigma^{*2}}\frac{M_N^*}{E_F^*}\right)\Pi_0(0)\right]^{-1}. \quad (23)$$

The fact that the same RPA denominator enters both expressions of $\Pi_{SS}$ and $\Pi_{VV}$, allows to infer the collective nature of $\Pi_{SS}$ from that of $\Pi_{VV}$. The quantity $\Pi_{VV}$, *i.e.*, the response to a probe which couples to the nucleon density fluctuations, is related to the nuclear compressibility. We introduce the incompressibility factor $K$ of the nuclear medium, defined as

$$K = 9\rho\frac{\partial^2\varepsilon}{\partial\rho^2} = 9\rho\frac{\partial\mu}{\partial\rho} = 9\rho\frac{\partial}{\partial\rho}\left(E_F^* + \frac{g_\omega^2}{m_\omega^2}\rho\right) = \frac{p_F}{E_F^*}\frac{\partial p_F}{\partial\rho} + \frac{M_N^*}{E_F^*} g_S^* \frac{\partial\bar{s}}{\partial\rho} + \frac{g_\omega^2}{m_\omega^2}. \quad (24)$$

The minimization (eq. (7)) establishes the dependence of $\bar{s}$ with respect to $\rho$. By taking its derivative, one gets

$$\frac{\partial\bar{s}}{\partial\rho} = -\frac{g_S^{*2}}{m_\sigma^{*2}}\frac{M_N^*}{E_F^*}. \quad (25)$$

It follows that

$$\frac{\Pi_{VV}(0)}{g_\omega^2} = -\frac{9\rho}{K}. \quad (26)$$

This relation is well known in the non-relativistic situation. We have shown that it also applies in the relativistic case with $\Pi_{VV}$ given above (eq. (23)). At low densities where relativistic effects are small, there is no distinction between the scalar and vector propagators and we have $\Pi_{SS}(0)/g_S^{*2} \simeq \Pi_{VV}(0)/g_\omega^2$. More generally the relation is

$$\frac{\Pi_{SS}(0)}{g_S^{*2}} = \left(\frac{M_N^*}{E_F^*}\right)^2 \frac{\Pi_{VV}(0)}{g_\omega^2}. \quad (27)$$

With our values of parameters the parenthesis represents a reduction of about 10% at $\rho_0$. The nuclear-physics information on the nuclear-matter compressibility leads to a value of $K$ in the range 200–300 MeV. This is close to the free Fermi-gas value, which is 230 MeV for $M_N^* = M_N$. Thus, the nuclear phenomenology which constraints the residual interaction at saturation density, also constraints scalar quantities, such as the scalar nuclear response and the QCD scalar susceptibility.

## 3 Numerical results and discussion

### 3.1 Sigma mass

The first consequence we wish to discuss is the problem of the density dependence of the scalar meson mass. Its explicit form (in the chiral limit) is given by eq. (8) and

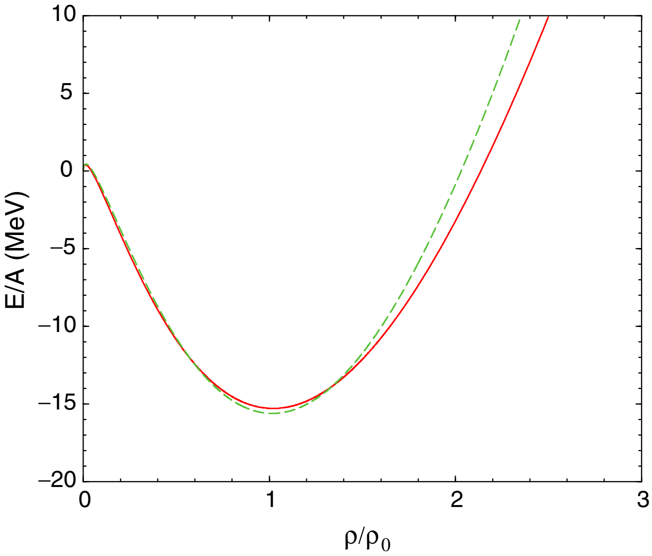


**Fig. 1.** Binding energy of nuclear matter. The dashed line corresponds to the set of parameters $g_\omega = 7$, $m_\sigma = 750$ MeV and $C = 0.85$ in the absence of the density dependence of the nucleon susceptibility. The full line corresponds to a set of parameters $g_\omega = 6.8$, $m_\sigma = 750$ MeV and $C = 1$ with an explicit field (density) dependence of the nucleon susceptibility as explained in the text.

we now come to the quantitative estimate. It is possible to get a first evaluation of the nucleon response, $\kappa_{NS}$, from the parameters of the QMC model, as given in [9]. It gives for the dimensionless parameter $C = (f_\pi^2/2M_N)\kappa_{NS}$ the value $C = 0.45$. Two independent parameters remain to be fixed: $g_S/m_\sigma$ and $g_\omega/m_\omega$. Since $g_S = M_N/f_\pi$ is given by our model and since we take for the omega mass the vacuum value $m_\omega = 783$ MeV, the parameters to be fixed are in fact $m_\sigma$ and $g_\omega$. In a first step we keep for the scalar potential, $V_0(s)$, only the quadratic part, namely, $V_0(s) = m_\sigma^2 s^2/2$, as in the original Walecka or QMC model which both ignore the chiral softening of the sigma mass. In such a case the saturation properties ($\rho_0 = 0.17\,\mathrm{fm}^{-3}$, $E/A = -15.3$ MeV) are obtained by taking $m_\sigma = 715$ MeV and $g_\omega = 7.47$. The corresponding incompressibility is $K = 260$ MeV and the effective nucleon mass at $\rho_0$ is $M_N^* = 760$ MeV.

If instead we introduce the full chiral $V_0(s)$, the effect of chiral restoration associated with the dropping of the sigma mass (see eq. (8)) and the accompanying discussion) destroys the saturation. In order to recover it, one has to increase the nucleonic susceptibility $\kappa_{NS}$ (or equivalently $C$). Indeed with value $C = 0.8$, and keeping the other parameters to the same values we find saturation at $\rho_0$. However, the binding energy $E/A = -17.8$ MeV and the incompressibility $K = 360$ MeV are too large. A slight readjustment of the parameters $C = 0.85$, $m_\sigma = 750$ MeV and $g_\omega = 7$ can solve the problem for the binding energy (see dashed curve of fig. 1) but the incompressibility remains too large (above 300 MeV). It can be brought to a more realistic value with a field dependence of the susceptibility. We take a simple linear one with a vanishing of

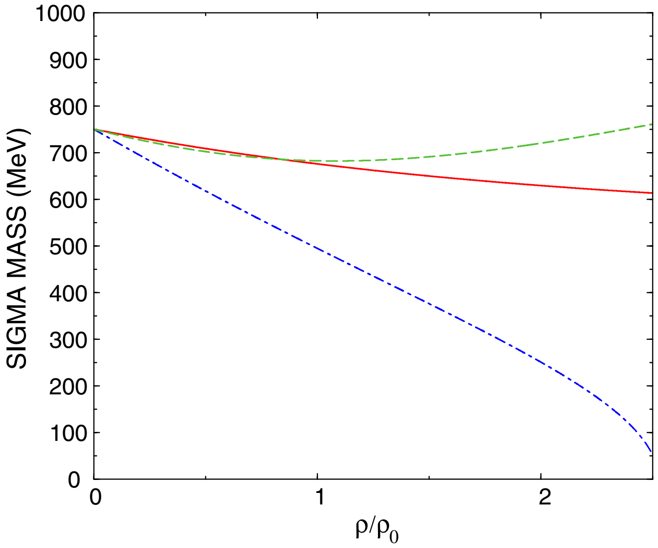


**Fig. 2.** Density evolution of the sigma mass. Dashed line: in the absence of the field (density) dependence of the nucleon susceptibility with values of the parameters $g_\omega = 7$, $m_\sigma = 750$ MeV and $C = 0.85$. Full line: with density dependence of the nucleon susceptibility with $g_\omega = 6.8$, $m_\sigma = 750$ MeV and $C = 1$. Dot-dashed line: it corresponds to the case where only the chiral softening is included, without the effect of the nucleon susceptibility.

$\kappa_{NS}$ at restoration ($\bar{s} = -f_\pi$). Hence in expression (6) of the mass we make the following replacement:

$$\kappa_{NS} \rightarrow \kappa_{NS}(\bar{s}) = \frac{\partial^2 M_N}{\partial \bar{s}^2} = \kappa_{NS}\left(1 + \frac{\bar{s}}{f_\pi}\right). \qquad (28)$$

In this case the set of parameters $g_\omega = 6.8$, $m_\sigma = 750$ MeV and $C = 1$, leads to correct saturation properties, with an incompressibility value $K = 270$ MeV (full curve of fig. 1) and $M_N^*(\rho_0) = 760$ MeV. Notice that in this case the corresponding value of the residual interaction $F_0$ at $\rho_0$ is practically zero ($F_0 \simeq 0.03$). This is due to a delicate cancellation between two large terms: the omega exchange and the in-medium modified scalar exchange, the magnitude of each term being of the order of 3.

For our main point which is the sigma-mass evolution, its general behaviour is, to a large extent, independent of the exact field dependence of the susceptibility. Figure 2 represents this evolution without (dot-dashed curve) and with the effect of the nucleonic scalar response, introducing or not its field dependence. The two curves with this nucleonic scalar response are rather flat. The nucleon reaction largely suppresses the strong softening due to chiral restoration which, if taken alone, would not be compatible with saturation properties. A similar conclusion was reached in ref. [6]. The stability of the sigma mass practically rules out the interpretation of the $2\pi$ production experiments in terms of the chiral dropping of the sigma mass. However, the more traditional interpretation of these data in terms of the modification of the $\pi\pi T$-matrix by the dressing of the pion lines by p-h bubbles [16] holds, at least for the photoproduction experiment [17]. In a forthcoming work we show that this effect also has a connection to chiral-symmetry restoration. We also demonstrate that this strong reshaping of the scalar strength in the nuclear medium does not affect the scalar $NN$ interaction.

### 3.2 Nuclear responses

Our second point concerns the nuclear response to a probe which couples to the nucleon density, scalar or vector, for which we have given the relativistic expressions (eqs. (19), (23)). At $\rho_0$ we have seen that the value of the incompressibility $K$ of nuclear matter lies in the range 200–300 MeV. This is close to the free Fermi-gas value, which is $K = 230$ MeV for $M_N^* = M_N$. In QMC where the effective mass does not differ so much from the free one, the experimental value of $K$ implies a small residual force at $\rho_0$. Our choice of parameters takes this constraint into account. This smallness of $F_0$ results from the accidental cancellation between omega and sigma exchanges. This delicate balance is upset with a change of the density. Several factors are responsible for this phenomenon. Firstly the relativistic factor $(M_N^*/E_F^*)$ lowers the sigma exchange while it enhances the omega one, by a density-dependent amount. However, this is not the main effect. The main one is due to the action of the nucleonic reaction $\kappa_{NS}$ which is responsible for the decrease of the scalar coupling constant with increasing density. The sigma effectively decouples from the nucleon at large density, leaving the repulsive omega interaction to dominate. The reverse occurs at smaller density with the increase of $g_S$. The sigma attraction then fully develops and dominates the repulsive omega exchange. Thus, with increasing density, the residual interaction turns from attraction into repulsion. Each component being large, the evolution is fast. For instance, with our parameters the resulting attraction is strong enough to produce a singularity of the polarization propagator at a density $\rho \simeq 0.6\rho_0$. We identify it with the spinodal instability. In the density region below the saturation one, the responses are collective with a resulting softening of the response. It is reflected as an enhancement in magnitude of the corresponding nuclear susceptibility, proportional to $\Pi_{SS}(0))$ or $\Pi_{VV}$. Above $\rho_0$, instead, collectivity hardens the response and decreases the susceptibility. Notice that the quantity $\Pi_{SS}(0))$ which incorporates the effective coupling constant $g_S^{*2}$ is suppressed at large densities not only by the collective RPA denominator but also by the dropping in the medium of $g_S^*$.

The density dependence of the residual interaction is well established [18,19]. It is small in the nuclear interior and strongly attractive at the nuclear periphery. On the other hand, the collective nature of the vector response is confirmed by data, from $(e,e')$ scattering, on the longitudinal response of various nuclei at small momenta. The charge response is a sum of a vector-isoscalar response and a vector-isovector one. Only the first part is relevant for our discussion. The data at small momenta (200–300 MeV/$c$) display a strong softening effect with respect to the free response. Alberico *et al.* [20] have attributed this feature to the collective attractive character, at the nuclear periphery hence at low density, of the isoscalar part of the charge response. The density dependence of the residual force with the evolution from attraction into repulsion with increasing density naturally follows in our description with sigma and omega exchange and with the incorporation of the nucleonic scalar response, necessary to favor the omega contribution relative to the sigma one with increasing density.

### 3.3 QCD susceptibilities

It is normal that the collective character of the nuclear response shows up in the QCD susceptibility, as the quark density fluctuations are coupled to the nucleonic ones. In ref. [2] we have derived, in the linear sigma model, the following relation between the in-medium QCD scalar susceptibility and the vacuum one:

$$\frac{\chi_S}{\chi_{S,vac}} = \frac{m_\sigma^2}{m_\sigma^{*2}}\left(1 - \frac{\Pi_{SS}(0)}{m_\sigma^{*2}}\right). \qquad (29)$$

For a quantitative evaluation we calculate the r.h.s. in our model with values of the parameters which are those given previously. The resulting density evolution is shown in fig. 3 in the case of a constant $\kappa_{NS}$. At $\rho_0$ the enhancement over the vacuum value is a factor of about 5. At lower density it becomes even larger due to the collectivity of the polarization propagator $\Pi_{SS}$, with the repulsive attractive p-h force which enhances its magnitude. At larger densities instead the p-h force suppresses $\Pi_{SS}$, which shows up in the gradual disappearance of the medium effects, as depicted in fig. 3. As $\Pi_{SS}$ approaches zero, only the effective-mass influence survives. It is somewhat model dependent through the density behaviour of $\kappa_{NS}$. With the introduction of a density dependence of $\kappa_{NS}$, a moderate enhancement (a factor of about 2) of the scalar susceptibility survives at density of the order of twice the nuclear-matter density.

It is interesting to contrast the evolution of the scalar susceptibility with that of the other QCD susceptibility, the pseudoscalar one. This last quantity is linked to the fluctuations of the quark pseudoscalar-isovector density. The question is wether the increase in magnitude of the scalar susceptibility due to the coupling to the nucleon-hole states leads to a convergence effect between the two susceptibilities, which would be a signal for chiral-symmetry restoration since the two susceptibilities become equal in the restored phase. In ref. [2] we have demonstrated that the evolution of the pseudoscalar susceptibility follows that of the quark condensate, with

$$\chi_{PS} = \frac{\langle \bar{q}q(\rho)\rangle}{m} \qquad (30)$$

which diverges in the chiral limit, as it should. The condensate on the r.h.s. is a function of the density and also of the quark mass. We calculate its value from eq. (14),

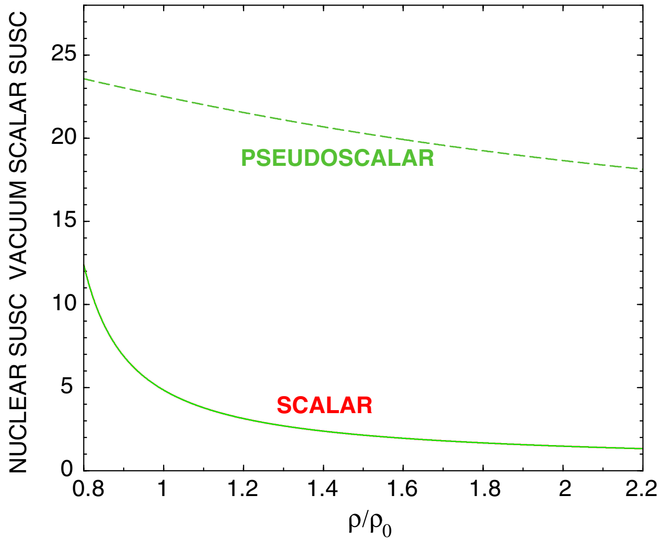


**Fig. 3.** Density evolution of the QCD susceptibilities normalized to the vacuum value of the scalar one calculated with the field dependence of the nucleon susceptibility. Full curve: scalar susceptibility. Dashed curve: pseudoscalar susceptibility.

at the physical value of the pion mass (quark mass). It is a smooth function of the density, as shown in fig. 3. Its nearly linear behaviour indicates the validity of the independent nucleon approximation. The spinodal instability which affects the scalar susceptibility does not influence the pseudoscalar one. At $\rho_0$ the two suceptibilities show an appreciable convergence effect. It is even larger at smaller density due to the spinodal unstability.

Our mean-field description ignores the role of the pion in the nuclear binding. Hence, the condensate evolution only incorporates the sigma influence as is clear from eq. (14). The role of the pion cloud is omitted, while it is known to be large. At $\rho_0$ the convergence effect will be more pronounced in the more complete approach which incorporates the pion. However, our conclusions about the stability of the sigma mass and the progressive decoupling of the quark density fluctuations from the nucleonic ones as the density increases will survive in the more complete approach.

## 4 Conclusion

In summary, our aim has been to evaluate the evolution with density of quantities linked to QCD or chiral symmetry: the quark condensate, the QCD scalar susceptibility and the sigma mass. For this we have worked in a relativistic chiral effective theory which describes reasonably well the saturation properties of nuclear matter. We have chosen a non-linear sigma model, implemented with the presence of a chiral invariant scalar field of mass $m_\sigma$ which provides the nuclear attraction. Adding the vector meson repulsion, we have the ingredients of quantum hadrodynamics and we have worked in the mean-field approximation. In order to counterbalance the chiral softening of the sigma mass in the medium, which prevents saturation from occurring, we have to incorporate in the theory the concept of the scalar response of the nucleon, as introduced in QMC. In order to reproduce the saturation properties, the cancellation effect has to be nearly complete and the evolution with density of the sigma mass becomes quite mild, which makes the interpretation of the $2\pi$ production experiments by the chiral dropping of the sigma mass problematic. However, introducing the quasiparticle character of the pion in the nuclear medium, one is able to reproduce the data, at least for the photoproduction experiment [17].

For the QCD scalar susceptibility, its magnitude is appreciably enhanced at normal saturation density due to the coupling of the quark scalar density fluctuations to the low-lying nuclear states. However, we have found that this effect does not develop further with increasing density and fades away as one gets closer to full restoration of chiral symmetry. The main reason is that the progressive decoupling of the sigma field from the nucleon induced by the nucleonic scalar response suppresses the effect of the nuclear excitations on the scalar susceptibility, which tends to become closer to its vacuum value.

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## Appendix A. Relativistic bare polarization propagators

First consider the bare first-order polarization propagator in the vector channel:

$$\frac{\Pi^0_{VV}(0)}{g_\omega^2} = -\lim_{\vec{q}\to 0}\int \frac{8\,\mathrm{d}^3p}{(2\pi)^3}\,\frac{\Theta(p_F-p)\,\Theta(|\vec{p}+\vec{q}|-p_F)}{E^*_{\vec{p}+\vec{q}}\,-\,E^*_p}. \tag{A.1}$$

We multiply the denominator and the numerator by $E^*_{\vec{p}+\vec{q}}\,+\,E^*_p$ which leads to

$$\frac{\Pi^0_{VV}(0)}{g_\omega^2} = -\lim_{\vec{q}\to 0}\int \frac{8\,\mathrm{d}^3p}{(2\pi)^3}\,(E^*_{\vec{p}+\vec{q}}\,+\,E^*_p) \times\frac{\Theta(p_F-p)\,\Theta(|\vec{p}+\vec{q}|-p_F)}{(\vec{p}+\vec{q})^2\,-\,p^2}. \tag{A.2}$$

Obviously, the factor $(E^*_{\vec{p}+\vec{q}}\,+\,E^*_p)$ in the integrand can be replaced by $2\,E^*_F$ since in the limit $\vec{q}\to 0$, $\vec{p}$ has to lie on the Fermi surface. It follows that

$$\frac{\Pi^0_{VV}(0)}{g_\omega^2} = -\frac{E^*_F}{M^*_N}\lim_{\vec{q}\to 0}\int \frac{8\,\mathrm{d}^3p}{(2\pi)^3}\,\frac{\Theta(p_F-p)\Theta(|\vec{p}+\vec{q}\,|-p_F)}{(\vec{p}+\vec{q})^2/2M^*_N - p^2/2M^*_N} \tag{A.3}$$

and the remaining integral is manifestly the nonrelativistic $\Pi_0(0)$. This establishes the second relation given in eq. (22). For each scalar vertex one gets an additional multiplying factor $M^*_N/E^*_F$. Thus, for one scalar vertex as is the case in the mixed term $\Pi^0_{SV}(0)$, the relativistic correction factor altogether disappears, hence the relation in eq. (22). The presence of two scalar vertices leads to the first relation for the pure scalar polarization propagator $\Pi^0_{SS}(0)$.

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